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Generation of Simplified Spacecraft Mathematical Models with Equivalent Dynamic Characteristics

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Equivalent spacecraft dynamic systems consisting of a number of single-degree-of-freedom models are described in the literature for one- and two-dimensional spacecraft dynamic models. In this paper a methodology is presented to generate simplified three-dimensional spacecraft dynamic models of complex spacecraft. The proposed methodology was employed for a preliminary coupled analysis of the CLUSTER spacecraft stack configuration. The simplified dynamic model produces equivalent dynamic response in terms of launcher spacecraft interface transient dynamic loads. The quality of the simplified dynamic representation is considered particularly useful for updating spacecraft design loads defined in launcher user manuals.

Nomenclature

f	= frequency
$f(t)$	= generalized force vector at structure base
I	= mass moment of inertia
$\boldsymbol{K}$	= stiffness matrix
$\boldsymbol{L}$	= matrix of modal participation factors
l	= position of modal center of gravity
$\boldsymbol{M}$	= mass matrix
m	= mass term
N	= number of structural degrees of freedom
n	= number of single-degree-of-freedom (SDOF) models in the equivalent spring-mass system
P	= first moment of mass
q	= vector of structural degrees of freedom
S	= static moment
v	= position vector of mass center of gravity
δ, η	= generalized coordinates associated with the rigid-body vectors and the elastic eigenvectors, respectively
$\boldsymbol{\Phi}_E$	= matrix of elastic eigenvectors
$\boldsymbol{\Phi}_R$	= matrix of rigid-body vectors (six vectors)

Superscripts

r	= reference point for rigid-body vectors
T	= transposed matrix

Subscripts

CoG	= mass center of gravity
c	= term resulting from eigenmode combination
E	= elastic modes
eff	= modal effective term
i	= internal degrees of freedom, summation index
R	= rigid-body modes
r	= interface degrees of freedom, rigid-body property
res	= residual inertia property
s, t	= arbitrary coordinate directions
x, y, z	= spatial coordinates

Introduction

IN order to effectively design spacecraft structures with low mass it is essential to define representative design loads. For the initial design of spacecraft structures the flight limit loads defined in the Launcher User Manual are employed with safety factors to determine the yield and ultimate strength. The loads defined in the Launcher User Manual are quasistatic accelerations applicable to the spacecraft center of gravity and are not adequate for the definition of dynamic responses in the spacecraft, e.g., on platform experiment mounts, antenna dishes, and flexible booms.

The structural response due to the excitation of the coupled launch-vehicle and spacecraft system can be obtained from a coupled-dynamic-loads analysis involving mathematical models of the launcher and the spacecraft. Figure 1 outlines the common practice in spacecraft design and indicates the position of the coupled analysis in the design process. In the coupled analysis the dynamic interaction of launcher and spacecraft is evaluated considering the transient nature of the loads. The dynamic loads acting on the spacecraft structure are dependent on the flight event; e.g., in the case of a coupled analysis with the ARIANE launcher three relevant load cases are included:

1) Longitudinal load cases:
   First-stage cutoff (EOF1)
   Second-stage cutoff (EOF2)
2) Lateral load case:
   maximum dynamic pressure (MDP)

Highly accurate coupled analysis results can be computed with a large three-dimensional (3D) spacecraft model in the composite model of launcher and spacecraft. However, the dynamic characteristics of a structure with an excitation at the base can be sufficiently represented by effective modal characteristics employing a limited number of spacecraft normal modes. Simplified models can be established, and the engineering and computational effort for the coupled analysis can be reduced drastically. As a result, employing simplified coupled-loads analysis, valuable information on representative design loads can be efficiently provided at an early design stage.

Generation of Equivalent Spacecraft Dynamic Systems

Equivalent spacecraft dynamic systems consist of a number of single-degree-of-freedom (SDOF) models.[1,2] The latter represent the effective characteristics of the dynamic eigenmodes of the spacecraft structure with respect to the reaction forces at the launcher spacecraft interface. The simplified models are obtained from the dynamic characteristics of the spacecraft structure hardmounted at the launcher spacecraft interface, and are defined in terms of the eigenvalues and the associated modal mass matrices.

The rigid-body contribution of the truncated eigenvectors must be added to the spring-mass system as residual inertia properties.

The equivalent spring-mass system is defined by the following set of parameters:

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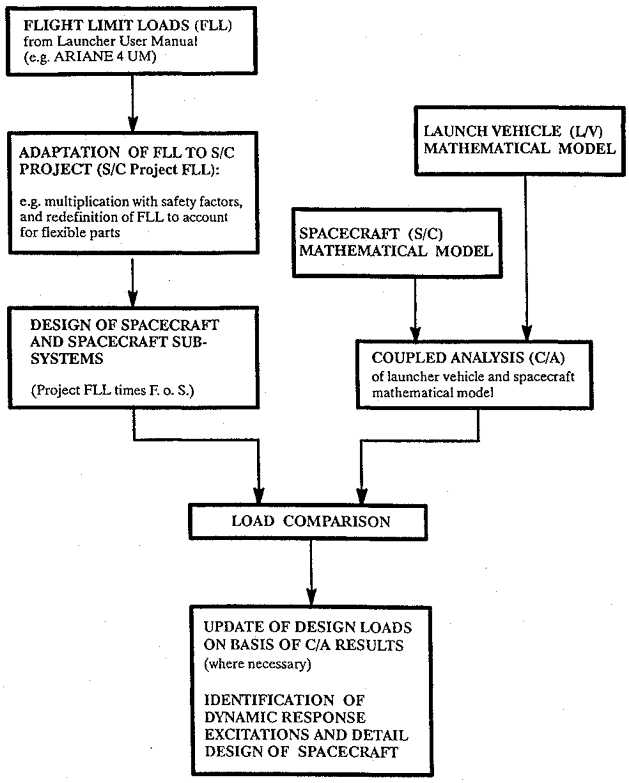


Fig. 1 Schematic of common practice in spacecraft design.

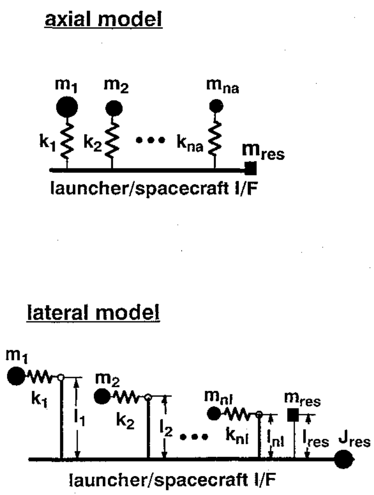


Fig. 2 Equivalent spring-mass systems for simplified axial and lateral models.

1) For every eigenmode considered in the model:
   eigenfrequency, f_i
   effective mass, m_i
   position of modal center of gravity, l_i (lateral models only)
2) residual mass (translational), m_{res}
3) additionally for lateral models:
   position of CoG of residual mass, l_{res}
   residual moment of inertia, I_{res}

Simplified equivalent spring-mass systems for respective axial and lateral models are sketched in Fig. 2.

Dynamics of Structures with Excitation at the Base

The spacecraft structure is represented by a dynamic system with N physical degrees of freedom partitioned into interface and internal degrees of freedom, denoted by the indices r and i respectively. Since the launcher spacecraft interface is assumed statically determinate, the interface degrees of freedom represent the rigid-body degrees of freedom of the spacecraft structure.

Employing the partitioned stiffness and mass matrices $\boldsymbol{K}$ and $\boldsymbol{M}$, the equation of motion for the undamped spacecraft structure can be written

$$\begin{bmatrix} M_{ii} & M_{ir} \\ M_{ir}^T & M_{rr} \end{bmatrix} \begin{Bmatrix} \ddot{q}_i \\ \ddot{q}_r \end{Bmatrix} + \begin{bmatrix} K_{ii} & K_{ir} \\ K_{ir}^T & K_{rr} \end{bmatrix} \begin{Bmatrix} q_i \\ q_r \end{Bmatrix} = \begin{Bmatrix} F_i \\ F_r \end{Bmatrix} \tag{1}$$

The modal transformation

$$[q] = [\,\Phi_R \;\; \Phi_E\,] \begin{Bmatrix} \delta \\ \eta \end{Bmatrix} \tag{2}$$

results in

$$\begin{bmatrix} M_{RR} & L^T \\ L & M_{EE} \end{bmatrix} \begin{Bmatrix} \ddot{\delta} \\ \ddot{\eta} \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & K_{EE} \end{bmatrix} \begin{Bmatrix} \delta \\ \eta \end{Bmatrix} = \begin{Bmatrix} f(t) \\ 0 \end{Bmatrix} \tag{3}$$

where

$$M_{RR} = \Phi_R^T M \Phi_R \qquad \text{(rigid-body mass matrix)}$$
$$M_{EE} = \Phi_E^T M \Phi_E \qquad \text{(generalized mass matrix)}$$
$$L = \Phi_E^T M \Phi_R \qquad \text{(matrix of modal participation factors)}$$
$$K_{EE} = \Phi_E^T K \Phi_E \qquad \text{(generalized stiffness matrix)}$$

The structural rigid-body mass matrix M_{RR} in Eq. (3) has the following structure:

$$M_{RR} = \left[\begin{array}{ccc|ccc} m_{xx} & 0 & 0 & 0 & P_{xy}^r & P_{xz}^r \\ 0 & m_{yy} & 0 & P_{yx}^r & 0 & P_{yz}^r \\ 0 & 0 & m_{zz} & P_{zx}^r & P_{zy}^r & 0 \\ \hline 0 & P_{yx}^r & P_{zx}^r & I_{xx}^r & I_{xy}^r & I_{xz}^r \\ P_{xy}^r & 0 & P_{zy}^r & I_{yx}^r & I_{yy}^r & I_{yz}^r \\ P_{xz}^r & P_{yz}^r & 0 & I_{zx}^r & I_{zy}^r & I_{zz}^r \end{array}\right] \tag{4}$$

where m_{ss} represents the rigid-body mass in the s direction, P_{st}^r the first moment of mass in the s direction about the t axis (static moment), and I_{ss}^r the mass moment of inertia about the s axis (second moment).

The rigid-body mass matrix M_{RR} is very helpful for checking the total mass in a finite-element model in each of the three coordinate directions. The mass center of gravity defined by the vector components v_x, v_y, and v_z can be calculated with respect to the reference point of the rigid-body vectors:

$$\begin{aligned} v_x &= P_{yz}^r/m_{yy} = -P_{zy}^r/m_{zz} \\ v_y &= P_{zx}^r/m_{zz} = -P_{xz}^r/m_{xx} \\ v_z &= P_{xy}^r/m_{xx} = -P_{yx}^r/m_{yy} \end{aligned} \tag{5}$$

In general the reference point of the rigid-body vectors Φ_R is different from the origin of the coordinate system. To identify the structural center of gravity the v_x, v_y, v_z vector locations need to be added to the reference point of the rigid vectors.

Dynamic mass concepts are particularly useful when considering the dynamic characteristics of a structure with an excitation at the base. The dynamic mass contribution of the ith elastic eigenmode to the rigid-body mass matrix M_{RR} is given by the modal mass matrix $M_{RR,i}$:

$$M_{RR,i} = \frac{1}{m_i} L_i^T L_i \tag{6}$$

where

$$m_i = \Phi_{Ei}^T M \Phi_{Ei}$$

is the generalized mass for the ith eigenmode, and

$$L_i = \Phi_{Ei}^T M \Phi_R$$

is the matrix of modal participation factors for the ith eigenmode. The contributions of the effective mass of all the fixed-free eigenmodes tend to the rigid-mass matrix[3] M_{RR}. As a result the sum of the effective dynamic mass for a large number of modes is of the form of the rigid-mass matrix M_{RR}:

$$\sum_{i=1}^{N} M_{RR,i} = \sum_{i=1}^{N} \frac{\Phi_R^T M \Phi_{Ei} \Phi_{Ei}^T M \Phi_R}{\Phi_{Ei}^T M \Phi_{Ei}} = M_{RR} \tag{7}$$

However, the dynamic mass contribution of each eigenmode does not essentially comply with such a form, and indeed the ith modal mass contribution to the rigid-mass matrix has the general form

$$M_{RR,i} = \frac{\Phi_R^T M \Phi_{Ei} \Phi_{Ei}^T M \Phi_R}{\Phi_{Ei}^T M \Phi_{Ei}} = \left[\begin{array}{ccc|ccc} X & X & X & X & X & X \\ & X & X & X & X & X \\ & & X & X & X & X \\ \hline & & & X & X & X \\ s & y & m & & X & X \\ & & & & & X \end{array}\right] \tag{8}$$

Physically the form of the matrix in Eq. (8) expresses the fact that reaction forces at the interface arise in all six directions from a modal response of eigenvectors with arbitrary shape. Therefore in principle the representation of a modal coordinate with a single physical degree of freedom is not possible on account of the ith modal mass contribution in Eq. (8). Only if the ith modal mass contribution is of the form

$$M_{RR} = \left[\begin{array}{ccc|ccc} X & 0 & 0 & 0 & X & X \\ & X & 0 & X & 0 & X \\ & & X & X & X & 0 \\ \hline & & & X & X & X \\ s & y & m & & X & X \\ & & & & & X \end{array}\right] \tag{9}$$

will a shift to the center of gravity and transformation to the principle inertia axes result in the identification of one physical coordinate to represent the modal coordinate.

In practice, although Eq. (8) is fully populated, modal mass contributions are often dominant only along one or two axes. Such an effect is easily identified by the magnitude of the diagonal terms. As a result, SDOF physical models, axial or lateral, can represent the ith modal mass contribution in Eq. (8). In the most general case, where there are significant reaction forces in all six interface degrees of freedom, a number of SDOF physical models are employed to represent the contributions in the various directions.

An application example for a complex spacecraft mathematical model is presented in detail later in the paper. It is noted that for a few modes two SDOF physical models are required to represent one modal coordinate—for example, mode 6 in Table 2 for the CLUSTER stack configuration.

Determination of the Dynamic Parameter for the SDOF Spring-Mass System

The selection of representative eigenmodes for the equivalent spring-mass model depends on:

1) The percentage of structural mass (e.g., rigid-body mass) that is represented by the main eigenmodes.

2) The maximum number of SDOF models that is specified by the launcher authority requirements for each load case.

Simplified Model Representation for Lateral Loads

The following parameters need to be defined for each eigenmode i that is represented in the equivalent spring-mass system:

1) Eigenfrequency f_i.

2) Effective mass $m_{\text{eff},i}$.

3) Position of modal mass center of gravity, l_i.

The contribution of the truncated eigenmodes to the spring-mass system, identified by residual terms of translational and rotational inertia masses, is determined by the following parameters:

1) Residual translational mass m_{res}.

2) Residual moment of inertia I_{res}.

3) Position of center of gravity, l_{res}, for residual mass m_{res}.

For the calculation of the parameters the effective mass matrix $M_{RR,i}$ of the ith eigenmode needs to be evaluated:

1) Effective mass as the diagonal element in the lateral s direction,

$$m_{\text{eff},i} = m_{ss}$$

2) Position of the modal center of gravity,

$$l_{i,s} = P_{st}/m_{ss}, \qquad l_{i,t} = -P_{ts}/m_{tt}$$

Ideally $l_{i,s} = l_{i,t}$.

The residual mass is the structural mass represented by the truncated eigenmodes, i.e., the difference between the rigid-body mass and the sum of all effective masses associated to the n eigenmodes included in the equivalent spring-mass system:

$$m_{\text{res}} = m_{\text{rr}} - \sum_{i=1}^{n} m_{\text{eff},i} \tag{10}$$

It is essential that the equivalent spring-mass system have the same static moment about the base point as the represented structure to comply with the static balance requirement. For static moment equilibrium the residual mass m_{res} is positioned at a distance l_{res} from the base point:

$$l_{\text{res}} = \frac{1}{m_{\text{res}}}\left(m_{\text{rr}} l_{\text{CoG}} - \sum_{i=1}^{n} m_{\text{eff},i} l_i\right) \tag{11}$$

where l_{CoG} is the distance of the structural center of gravity from the base point.

It is essential that the moments of inertia of all masses of the spring-mass system be equal to the moment of inertia of the represented 3D structure. Then the residual moment of inertia I_{res} can be computed from

$$I_{\text{res}} = I_{\text{base}} - \sum_{i=1}^{n} m_{\text{eff},i} l_i^2 - m_{\text{res}} l_{\text{res}}^2 \tag{12}$$

where I_{base} is the moment of inertia of the 3D finite-element model about the base point.

Simplified Model Representation for Axial Loads

The simplified dynamic model in the axial direction is established similarly to the lateral model representation. The following parameters need to be defined for each eigenmode i that is represented in the equivalent spring-mass system:

1) Eigenfrequency f_i.

2) Effective mass $m_{\text{eff},i}$.

The effective mass $m_{\text{eff},i}$ for the ith eigenmode is extracted from the diagonal element in the axial s direction of the effective-mass matrix $M_{RR,i}$:

$$m_{\text{eff},i} = m_{ss}$$

The contribution of the truncated eigenmodes to the spring-mass system in the axial direction is determined in terms of the total axial mass and all effective modal masses of the n eigenmodes that are included in the equivalent spring-mass system, as in Eq. (10).

Combination of Adjacent Eigenmodes

The maximum number of SDOF models in an equivalent spring-mass system can be greatly limited by launcher-authority requirements, e.g., 10 eigenmodes for a preliminary coupled analysis with the Ariane launcher. As a result, when establishing an equivalent dynamic system for a spacecraft structure, the problem arises of selecting 10 modes from a large set of eigenmodes for the equivalent spring-mass system that is provided to the launcher authority.

In order to increase modal representation in a limited number of SDOF models, a combination of adjacent eigenmodes can be performed. Adjacent eigenmodes are defined as those that are very close in frequency (within about 1 Hz) and that have effective masses associated with the same degree of freedom. The method described hereafter was established by engineering judgement, and is not based on rigorous mathematics. However, the application of this method is useful for models employed in simplified coupled analysis as presented for the 4 CLUSTER spacecraft stack configuration.

When combining two eigenmodes into one single reduced eigenmode with respect to the base force effect, parameters describing SDOF models in the equivalent spring-mass system are redefined by summation and averaging:

1) Combined eigenfrequency:

$$f_c = f_i + \frac{m_{i+1}}{m_i + m_{i+1}} \Delta f$$

$$\Delta f = f_{i+1} - f_i$$

2) Combined effective mass:

$$m_c = m_i + m_{i+1}$$

3) Combined effective length (modal center of gravity):

$$l_c = l_i + \frac{m_{i+1}}{m_i + m_{i+1}} \Delta l$$

$$\Delta l = l_{i+1} - l_i$$

where

f_i, f_{i+1} = lower and upper eigenfrequency of eigenmodes to be combined,
m_i, m_{i+1} = effective masses associated with the eigenmodes of eigenfrequencies f_i and f_{i+1}, respectively
l_i, l_{i+1} = positions of modal centers of gravity

Application Example: 4 CLUSTER Spacecraft Stack

The 4 CLUSTER spacecraft stack configuration is presented as an application example for simplified dynamic representation and preliminary coupled dynamic analysis. The procedure established for generating equivalent spring-mass systems is employed for a complex spacecraft mathematical model and is fully validated by a transient-response analysis employing the 3D structural model.

The finite-element model of the 4 CLUSTER spacecraft stack configuration is presented in Fig. 3. The lateral directions are described by the x and y coordinates, and the longitudinal direction by the z coordinate. The main characteristics of the 4 CLUSTER spacecraft stack with respect to its mass and geometrical properties are given in Table 1.

Table 1 Main characteristics of the four CLUSTER stack configuration

Total mass:
$m_{\text{tot}} = 4{,}131.0$ kg
Position of center of gravity:
$x_{\text{COG}} = 0.000$ m
$y_{\text{COG}} = 0.000$ m
$z_{\text{COG}} = 1.970$ m
Static moments (base):
$S_y = 8{,}138.1$ kg-m
$S_x = 8{,}138.1$ kg-m
Mass moments of inertia (CoG):
$I_{xx} = 7{,}051.3$ kg-m^2
$I_{yy} = 7{,}051.3$ kg-m^2
$I_{zz} = 3{,}838.0$ kg-m^2
Mass moments of inertia (base):
$I'_{xx} = 23{,}078.3$ kg-m^2
$I'_{yy} = 23{,}078.3$ kg-m^2
$I'_{zz} = 3{,}838.0$ kg-m^2

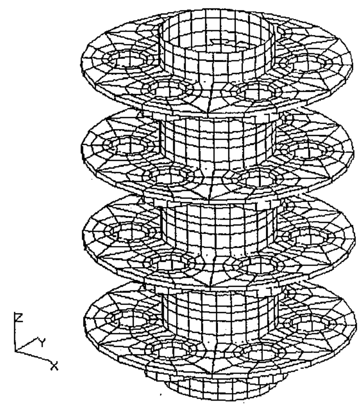


Fig. 3 Finite-element model of 4 CLUSTER spacecraft stack configuration.

Selection of Representative Eigenmodes

In the first analysis step the eigenfrequencies and eigenvectors of the 4 CLUSTER spacecraft stack were determined employing the 3D structural mathematical model. The effective modal masses of the main diagonal of the modal mass matrix (three translational and three rotational) are listed in Table 2. Inspecting the translational effective masses, indicated by m_x, m_y and m_z, the eigenmodes with predominant deformations either in the lateral x and y directions or in the longitudinal z direction are identified. The magnitude of the effective modal mass is employed to identify the importance of the modes and the coordinate direction. Significant effective modal masses have been boldfaced in Table 2.

In Table 2 the predominant direction of the eigenmodes of the 4 CLUSTER spacecraft stack configuration is classified by

A for longitudinal (axial) direction
L_x for lateral x direction
L_y for lateral y direction
T for torsional eigenmodes about the longitudinal z axis

As can be seen from the diagonal terms of the translational effective masses, many eigenmodes of the 4 CLUSTER spacecraft stack configuration show high coupling between one lateral and the longitudinal direction. These eigenmodes are indicated by a classification code, A for axial and L_x or L_y for lateral. In such cases the eigenmodes are selected for both longitudinal and lateral equivalent spring-mass systems.

Significant effective-mass moments of inertia I_x and I_y are associated with eigenmodes with predominant deformations in the lateral y and x directions, respectively. However, the rotational effective-mass terms I_{xi} and I_{yi} of the ith elastic eigenmode are not employed for the determination of lateral SDOF spring-mass systems, as explained later when describing the derivation of the position of the modal center of gravity.

The torsional eigenmodes about the longitudinal z axis can be identified by high effective-mass moments of inertia I_z. In simplified coupled analyses for payloads launched by the Ariane launcher the torsional spacecraft eigenmodes are generally not excited by flight events and are neglected in the coupled analysis.

The eigenmodes selected for the longitudinal and the lateral directions are presented in Table 3.

Determination of the Center of Gravity for Effective Modal Masses

The equivalent spring-mass system model parameters to be used for the simplified longitudinal coupled analysis can be evaluated from the information presented in Table 2. It can be determined from Table 4 that the offset of the axial mass given by the effective lengths v_{xy} and v_{yx} is negligible. However, for the lateral equivalent spring-mass system the position of the effective modal mass, i.e., the distance of the effective mass from the interface plane between spacecraft and launch vehicle, needs to be determined. The effective length identifies the contribution of the inertia forces of the effective modal mass to the bending moment about the interface point. As a

Table 2 4 CLUSTER spacecraft stack configuration: eigenfrequencies and modal effective masses

			Modal effective masses					
			Translational masses, kg			Rotational masses, kg-m^2		
Mode		f, Hz	m_x	m_y	m_z	I_x	I_y	I_z
1	L_y	12.89	611.3	**1,754.8**	0.0	16,443.0	5,729.0	0.0
2	L_x	12.89	**1,755.5**	611.7	0.0	5,729.6	16,444.6	0.0
3	T	24.90	0.1	0.1	0.0	0.1	0.1	3,110.4
4	A	33.30	13.9	11.7	**2,305.0**	7.7	9.1	0.0
5	L_y	33.65	91.9	**343.8**	9.8	260.9	69.6	1.0
6	L_x, A	33.84	**343.0**	84.6	**127.0**	62.7	253.8	0.1
7	L_y	36.65	10.6	**70.0**	1.5	26.5	4.3	0.0
8	L_x, A	36.81	**45.4**	5.3	**66.3**	1.3	15.2	0.0
9	L_y	36.95	11.7	**44.5**	1.4	2.4	1.3	0.0
10	A	37.04	0.1	2.5	**69.6**	1.1	1.3	0.0
11	L_x, A	37.16	**53.0**	12.0	**22.5**	2.8	10.9	0.0
12		37.40	19.3	5.0	18.2	0.1	0.4	0.0
13	A	39.57	4.3	2.2	**600.0**	3.7	6.6	0.0
14	L_y	39.88	10.9	**38.6**	1.4	121.8	32.1	0.3
15	L_x, A	40.32	**37.5**	9.0	**51.5**	30.2	121.3	0.1
16	L_x	41.37	**40.6**	10.1	0.1	1.3	5.6	0.0
17	L_y	41.68	11.6	**50.1**	0.0	11.6	2.8	0.0
18	L_x	42.77	**18.5**	4.9	0.0	0.7	2.6	0.0
19	L_y	43.20	5.8	**25.0**	0.0	4.1	1.0	0.0
20		43.38	4.9	1.2	0.0	0.0	0.0	0.0
21		43.85	1.4	6.0	0.0	0.1	0.0	0.0
22		47.49	3.8	1.0	0.0	3.9	18.2	0.0
23		48.02	1.8	11.2	0.0	19.7	4.5	0.1
24		52.00	0.0	0.2	0.0	0.0	0.0	223.1
25	L_y	54.87	10.7	**24.1**	0.0	11.0	5.0	13.0
26	L_y	55.07	109.6	**371.3**	0.8	161.8	47.1	0.0
27	L_x	55.29	**373.9**	91.4	0.6	40.2	161.9	0.0
28	A	55.48	47.9	10.5	**21.3**	4.4	20.7	0.0
29		55.80	4.6	28.1	4.7	12.2	2.0	0.1
30		55.84	0.9	13.7	3.8	5.9	0.4	2.5
31	A	56.65	0.1	0.2	**19.2**	0.2	0.0	1.2
32		57.11	0.1	0.1	0.4	0.0	0.0	4.1
33	T	58.07	0.4	0.5	0.1	0.1	0.0	204.1
34		61.28	0.1	0.2	0.0	0.0	0.0	3.8
35		67.43	0.0	0.0	0.0	0.0	0.0	67.0
36	T	70.49	0.2	0.2	0.0	0.1	0.1	125.4
37	L_y	71.44	32.9	**69.5**	0.0	9.5	4.5	0.4
38	L_x	71.94	**50.8**	29.9	0.0	4.1	7.0	0.0
39		72.26	5.5	1.8	0.1	0.3	1.2	0.0
40	L_y	73.45	23.5	**42.8**	0.0	3.1	1.9	0.3
41	L_x	73.86	**39.7**	21.3	0.0	1.7	3.8	0.1
42		74.08	0.0	0.0	0.0	0.1	0.0	0.0
43		74.97	0.0	0.0	0.1	0.0	2.0	0.0
44		76.09	0.0	1.7	0.0	3.8	0.1	0.0
45		80.34	15.8	0.5	0.1	0.1	3.2	0.0
46		81.18	2.4	17.5	0.1	3.5	0.4	0.0
47	L_x	82.84	**81.7**	2.7	0.1	0.3	11.2	0.0
48	L_y	83.54	2.6	**69.0**	0.0	9.7	0.4	0.0
49		86.71	0.0	0.0	2.1	0.0	0.1	0.0
50		87.31	85.3	13.9	0.0	0.6	3.6	0.0
51		87.94	15.0	78.5	0.3	3.2	0.8	0.1
52		88.05	0.7	6.4	1.8	1.0	0.0	0.0
53	A	91.90	0.0	0.0	**400.7**	0.0	0.0	0.0
54		92.62	0.9	0.0	0.0	0.0	1.9	0.1
55		93.59	0.1	2.5	0.0	3.1	0.1	0.0
56		96.82	11.0	1.6	0.3	0.9	4.3	0.3
57		97.72	3.0	16.9	0.9	5.4	0.9	0.0
58		98.32	0.0	0.1	12.3	0.2	0.2	0.0
59		99.35	0.0	0.0	0.4	0.2	0.1	0.0
60		101.62	2.5	0.6	0.0	1.4	6.3	0.0
⋮		⋮	⋮	⋮	⋮	⋮	⋮	⋮
99		162.26	0.0	0.0	0.0	0.0	0.0	3.6
100		163.68	0.0	0.1	0.0	0.0	0.0	5.6
Sum of effective masses:			4,060.8	4,061.2	3,916.6	23,041.0	23,042.0	3,776.6

Table 3 Equivalent spring-mass systems of 4 CLUSTER stack configuration

	Lateral x			Lateral y			Longitudinal z	
Mode	f, Hz	$m_{\mathrm{eff},i}$, kg	l_i, m	f, Hz	$m_{\mathrm{eff},i}$, kg	l_i, m	f, Hz	$m_{\mathrm{eff},i}$, kg
1	12.89	1,755.5	3.061	12.89	1,754.8	3.061	33.30	2,305.0
2	33.84	343.0	0.861	33.65	343.8	0.871	33.84	127.0
3	36.81	45.4	0.542	36.65	70.0	0.626	36.81	66.3
4	37.16	53.0	0.467	36.95	44.5	0.283	37.04	69.6
5	40.32	37.5	−1.813	39.88	38.6	−1.746	37.16	22.5
6	41.37	40.6	0.364	41.68	50.1	0.486	39.57	600.0
7	42.77	18.5	0.373	43.20	25.0	0.408	40.32	51.5
8	55.29	373.9	0.660	54.87	24.1	0.680	55.48	21.3
9	71.94	50.8	0.370	55.07	371.3	0.658	56.65	19.2
10	73.86	39.7	0.297	71.44	69.5	0.370	91.90	400.7
11	82.84	81.7	0.353	73.45	42.8	0.277		
12				83.54	69.0	0.390		

Table 4 Position of modal center of gravity for 4 CLUSTER spacecraft stack configuration

			Effective length, m				
Mode		f, Hz	$v_{xy} = \frac{P_{yz}}{m_{yy}}$	$v_{yx} = \frac{-P_{xz}}{m_{xx}}$	$v_{zx} = \frac{P_{xy}}{m_{xx}}$	$v_{zy} = \frac{-P_{yx}}{m_{yy}}$	$l_i = \frac{v_{zx}+v_{zy}}{2}$
1	L_y	12.89	−0.001	−0.002	3.061	3.061	3.061
2	L_x	12.89	0.000	0.000	3.061	3.061	3.061
⋮							
5	L_y	33.65	−0.054	−0.105	0.870	0.871	0.871
6	L_x	33.84	0.041	−0.020	0.860	0.861	0.861
7	L_y	36.65	0.011	0.029	0.636	0.616	0.626
8	L_x	36.81	−0.009	0.003	0.578	0.505	0.542
9	L_y	36.95	0.006	0.011	0.332	0.234	0.283
⋮							
11	L_x	37.16	−0.004	0.002	0.454	0.480	0.467
⋮							
14	L_y	39.88	0.094	0.177	−1.715	−1.777	−1.746
15	L_x	40.32	−0.082	0.040	−1.799	−1.827	−1.813
16	L_x	41.37	−0.025	0.012	0.372	0.357	0.364
17	L_y	41.68	0.009	0.019	0.492	0.481	0.486
18	L_x	42.77	−0.063	0.033	0.378	0.368	0.373
19	L_y	43.20	−0.003	−0.006	0.409	0.407	0.408
⋮							
25	L_y	54.87	0.734	1.103	0.683	0.676	0.680
26	L_y	55.07	0.008	0.014	0.656	0.660	0.658
27	L_x	55.29	0.021	−0.010	0.658	0.663	0.660
⋮							
37	L_y	71.44	−0.075	−0.109	0.371	0.369	0.370
38	L_x	71.94	0.018	−0.014	0.370	0.370	0.370
⋮							
40	L_y	73.45	−0.085	−0.115	0.285	0.268	0.277
41	L_x	73.86	0.053	−0.039	0.308	0.285	0.297
⋮							
47	L_x	82.84	0.035	−0.006	0.371	0.336	0.353
48	L_y	83.54	−0.021	−0.111	0.406	0.375	0.390

result the position of the modal center of gravity is important for identifying the sign of the rotation, i.e., a positive effective length indicates a clockwise rotation, and a negative length an anticlockwise rotation. The terms I_{xi} and I_{yi} of the ith elastic eigenmode, which are in general related to the translational effective mass $m_{\mathrm{eff},i}$ and the position of the modal center of gravity (effective length l_i) by the equation

$$I_{x(y)i} = m_{\mathrm{eff},i} l_i^2$$

are unsuited for this purpose. Therefore the static moment of each effective mass has been employed to determine the sign of the effective length l_i:

$$S_{x(y)i} = m_{\mathrm{eff},i} l_i$$

By inspecting Table 4 with respect to the effective lengths v_{zx} and v_{zy}, since the z axis is the longitudinal axis for the CLUSTER stack configuration as indicated in Fig. 3, it can be seen that the two values do not precisely match for higher lateral modes. Therefore, a mean value for the effective length l_i of the ith eigenmode, $i > 2$, was calculated from v_{zx} and v_{zy}:

$$l_i = \frac{v_{zx} + v_{zy}}{2}$$

The effective lengths calculated for all lateral eigenmodes in this way can be found by considering the data presented in Tables 2 and 4.

Combination of Adjacent Modes

In Table 3 are presented the eigenmodes of the 4 CLUSTER spacecraft stack configuration that were selected for the equivalent spring-mass systems in the lateral and longitudinal directions. The maximum number of SDOF models for each A, L_x, and L_y equivalent spring-mass system is limited to 10 for the preliminary Ariane launcher coupled analysis.

For the longitudinal direction this requirement is adequately covered by selecting the 10 main axial eigenmodes. However, in both lateral directions more than 10 representative eigenmodes are significant and need to be represented. In order to increase the modal representation in a limited number of SDOF models, close eigenmodes were combined as described above.

For the lateral x direction eigenmodes 6 and 7 were combined to the reduced eigenmode 6*:

$$\begin{Bmatrix} f_6^* \\ m_6^* \\ l_6^* \end{Bmatrix} = \begin{Bmatrix} 41.81 \text{ Hz} \\ 59.1 \text{ kg} \\ 0.367 \text{ m} \end{Bmatrix}$$

For the lateral y direction eigenmodes 6 and 7 were combined to the reduced eigenmode 6*:

$$\begin{Bmatrix} f_6^* \\ m_6^* \\ l_6^* \end{Bmatrix} = \begin{Bmatrix} 42.19 \text{ Hz} \\ 75.1 \text{ kg} \\ 0.460 \text{ m} \end{Bmatrix}$$

and eigenmodes 8 and 9 to the reduced eigenmode 7*:

$$\begin{Bmatrix} f_7^* \\ m_7^* \\ l_7^* \end{Bmatrix} = \begin{Bmatrix} 55.06 \text{ Hz} \\ 395.4 \text{ kg} \\ 0.659 \text{ m} \end{Bmatrix}$$

Residual Inertia Terms

The residual inertia terms correspond to the reactions, at the base, of the truncated dynamic modes. As described earlier, the residual inertia terms are considered in the equivalent spring-mass system according to their rigid-body contribution.

For the lateral load case the residual mass, the residual inertia (located at the base), and the position of the center of gravity of the residual mass from the base need to be determined.

1) *Residual mass:*

$$m_{\text{res}} = m_{rr} - \sum_{i=1}^{10} m_{\text{eff},i}$$

Employing the values in Table 5 for the effective masses results in:

$$m_{\text{res},x} = 4131.0 - 2839.6 = 1291.4 \text{ kg}$$

$$m_{\text{res},y} = 4131.0 - 2903.5 = 1227.5 \text{ kg}$$

2) *Position of the residual center of gravity:*

$$l_{\text{res}} = \frac{1}{m_{\text{res}}}\left(m_{rr} l_{\text{CoG}} - \sum_{i=1}^{10} m_{\text{eff},i} l_i\right)$$

Employing the values in Table 5 for the static moment of each effective mass with respect to the base and noticing that the term $m_{rr} l_{\text{CoG}}$ equals the static moment S_x or S_y in Table 1 results in

$$l_{\text{res},x} = \frac{8138.1 - 5978.2}{1291.4} = 1.673 \text{ m}$$

$$l_{\text{res},y} = \frac{8138.1 - 6019.4}{1227.5} = 1.641 \text{ m}$$

Table 5 Equivalent spring-mass systems of four CLUSTER stack configuration provided to launcher authority

	Lateral x				
Mode	f, Hz	$m_{\text{eff},i}$, kg	l_i, m	S_{yi}, kg-m	I_{yi}, kg-m^2
1	12.89	1,755.5	3.061	5,373.6	16,448.5
2	33.84	343.0	0.861	295.3	254.3
3	36.81	45.4	0.542	24.6	13.3
4	37.16	53.0	0.467	24.8	11.6
5	40.32	37.5	−1.813	−68.0	123.3
6	41.81	59.1	0.367	21.7	8.0
7	55.29	373.9	0.660	246.8	162.9
8	71.94	50.8	0.370	18.8	6.9
9	73.86	39.7	0.297	11.8	3.5
10	82.84	81.7	0.353	28.8	10.2
Total		2,839.6		5,978.2	17,042.5
m_{res}		1,291.4			
S_{res}				2,159.9	
l_{res}			1.673		
I_{res}					2,421.3

	Lateral y					Longitudinal z	
Mode	f, Hz	$m_{\text{eff},i}$, kg	l_i, m	S_{xi}, kg-m	I_{xi}, kg-m^2	f, Hz	$m_{\text{eff},i}$, kg
1	12.89	1,754.8	3.061	5,371.4	16,442.0	33.30	2,305.0
2	33.65	343.8	0.871	299.4	260.8	33.84	127.0
3	36.65	70.0	0.626	43.8	27.4	36.81	66.3
4	36.95	44.5	0.283	12.6	3.6	37.04	69.6
5	39.88	38.6	−1.746	−67.4	117.7	37.16	22.5
6	42.19	75.1	0.460	34.5	15.9	39.57	600.0
7	55.06	395.4	0.659	260.6	171.7	40.32	51.5
8	71.44	69.5	0.370	25.7	9.5	55.48	21.3
9	73.45	42.8	0.277	11.9	3.3	56.65	19.2
10	83.54	69.0	0.390	26.9	10.5	91.90	400.7
Total		2,903.6		6,019.4	17,062.4		3,683.1
m_{res}		1,227.5					447.9
S_{res}				2,118.7			
l_{res}			1.641				
I_{res}					2,711.9		

3) *Residual moment of inertia:*

$$I_{\text{res}} = I_{\text{base}} - \sum_{i=1}^{10} m_{\text{eff},i} l_i^2 - m_{\text{res}} l_{\text{res}}^2$$

Employing the values in Table 5 for the mass moment of inertia of each effective mass and the residual mass with respect to the base, and noticing further that I_{base} equals the value of I'_{xx} or I'_{yy} in Table 1, we have

$$I_{\text{res},x} = 23078.3 - 17042.5 - 1291.4 \times 1.673^2 = 2421.3 \text{ kg-m}^2$$
$$I_{\text{res},y} = 23078.3 - 17062.4 - 1227.5 \times 1.641^2 = 2711.9 \text{ kg-m}^2$$

For the axial simplified model representation only the residual mass is required:

$$m_{\text{res}} = m_{rr} - \sum_{i=1}^{10} m_{\text{eff},i}$$

Employing the values in Table 5 for the effective masses in the longitudinal direction results in

$$m_{\text{res},z} = 4131.0 - 3683.1 = 447.9 \text{ kg}$$

The reduced model for the simplified dynamic representation is presented in Table 5 including all residual inertia terms.

Validation of the Equivalent Spacecraft Dynamic Systems

The equivalent spring-mass systems of the 4 CLUSTER spacecraft stack configuration were provided to the launcher authority for employment in a coupled analysis with the dynamic model of the Ariane launcher. The results of the coupled analysis were provided in terms of accelerations and forces and/or moments at the interface between the launcher and the spacecraft for the following load cases:

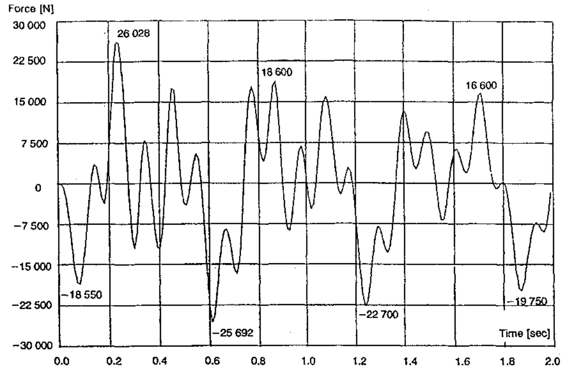
1) Maximum dynamic pressure (MDP):
   Lateral and rotational accelerations
   Lateral shear forces and bending moments
2) End of flight (EOF):
   Longitudinal accelerations
   Normal forces in longitudinal direction

The interface forces and moments computed in the launcher spacecraft coupled analysis employing the 3D simplified dynamic models of the CLUSTER stack configuration are presented in Figs. 4 and 5. Peak values of interface reactions are identified in the plots to facilitate data comparison.

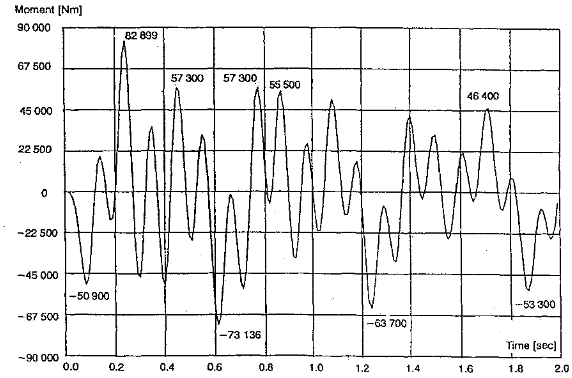
The interface accelerations provided by the launcher authority were employed to perform a transient-response analysis with the 3D structural model of the CLUSTER stack configuration excited at the base. Essentially this is the final step in the preliminary coupled analysis procedure to derive dynamic response and loads in the spacecraft.

The quality of the equivalent spring-mass system, i.e., the representation of the dynamic characteristics of the 3D structural model by the simplified model, was checked by comparing the reaction forces of the 3D structural model of the CLUSTER stack configuration with those of the equivalent spring-mass system for the coupled analysis. Ideally the base reactions of the simplified model will match the corresponding reactions of the 3D model for spacecraft excitation with the coupled analysis interface accelerations.

In Figs. 6 and 7 the interface forces and moments calculated with the 3D structural model of the 4 CLUSTER spacecraft stack configuration are graphically presented and their peak values identified. Comparing these curves with those in Figs. 4 and 5, the shape of the curves and the peak values match very well. Differences are identified in the calculation of the base bending moment. These might arise from the limited number of modes and from inaccuracies in the determination of effective lengths for the equivalent lateral spring-mass system.

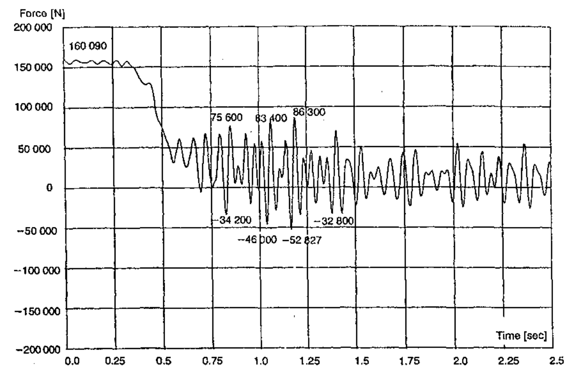


a) Base shear force

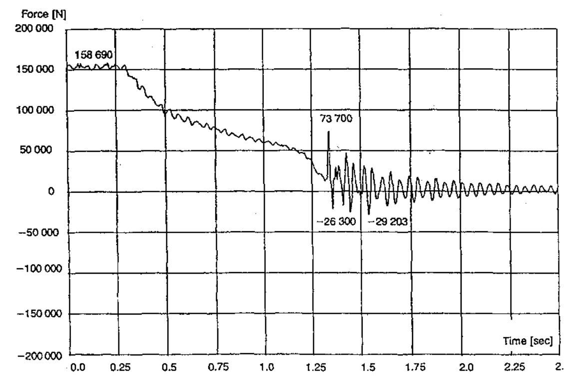


b) Base bending moment

Fig. 4 Stack configuration of 4 CLUSTER spacecraft: interface reactions for load case MDPX (output of Arianespace coupled-loads analysis employing the simplified models).

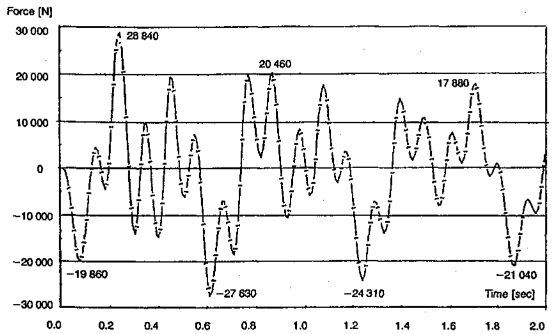


a) Base normal force (EOF1)

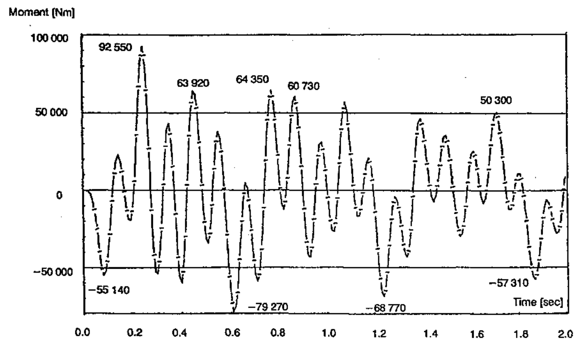


b) Base normal force (EOF2)

Fig. 5 Stack configuration of 4 CLUSTER spacecraft: interface reactions for load cases EOF1 and EOF2 (output of Arianespace coupled-loads analysis employing the simplified models).

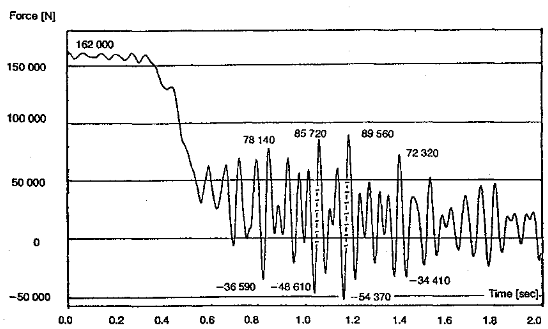


a) Base shear force

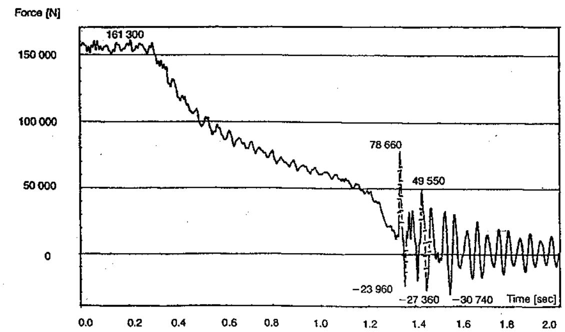


b) Base bending moment

Fig. 6 Stack configuration of 4 CLUSTER spacecraft: interface reactions for load case MDPX (output of transient-response analysis employing the 3D structural model).



a) Base normal force (EOF1)



b) Base normal force (EOF2)

Fig. 7 Stack configuration of 4 CLUSTER spacecraft: interface reactions for loadcases EOF1 and EOF2 (output of transient-response analysis employing the 3D structural model).

Conclusions

The generation of simplified equivalent spacecraft dynamic systems is based upon the assumption that the form of the modal mass matrices complies with the form of the structural rigid-body mass matrix. However, this is not the case in practice for spacecraft design applications. As a result, in principle the representation of a modal coordinate with a single physical degree of freedom is not possible. However, since in practice the modal mass contribution is dominant only along one or two axes, the modal mass of each mode is often adequately represented with a SDOF axial or lateral physical model. In the most general case where significant reaction forces are present in all six interface degrees of freedom, a number of SDOF physical models can be employed to represent the contributions in the various directions. In the case that the maximum number of SDOF models for the equivalent spring-mass system is limited by launcher-authority requirements, adjacent eigenmodes can be combined to a single reduced eigenmode representation by summation and averaging. In this way the modal data contained in the simplified models can be increased. The application of the method to the generation of the equivalent spacecraft dynamic sytems for the CLUSTER simplified coupled analysis produced accurate results of high practical importance.

The simplified dynamic model produces equivalent dynamic responses with respect to the time history of launcher spacecraft interface loads, e.g., shape and magnitude. The quality of the simplified dynamic representation is considered particularly useful for updating the spacecraft design loads defined in launcher users' manuals, and to obtain at an early stage the preliminary dynamic response and loads in the satellite.

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